



## VIII Astronomy Olympiad Pre-Departure Camp, 5<sup>th</sup> – 9<sup>th</sup> Nov, 2006

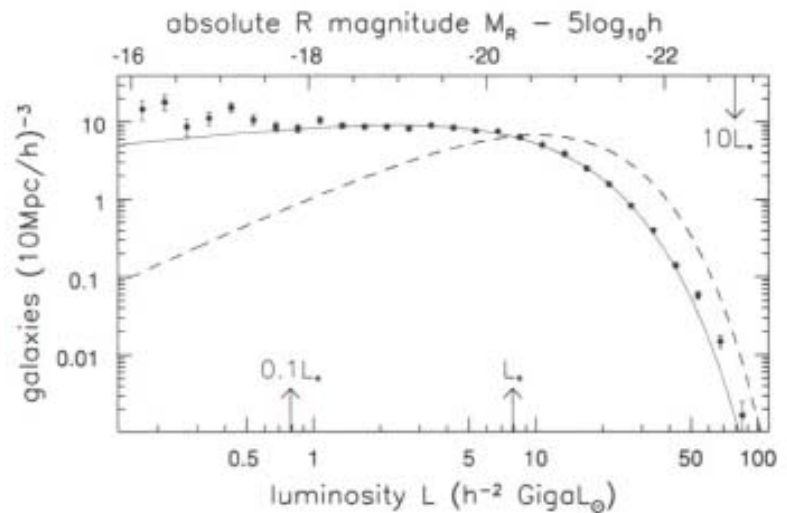
Sponsored by HBCSE – TIFR, Mankhurd, Mumbai

### Theory Test – I (Time: 2<sup>h</sup>; Mk: 50)

1. The black holes which power quasars have masses typically about  $10^8 M_{\text{sun}}$ . The Schwarzschild radius  $R$  of a black hole is proportional to its mass  $M$ , with the Schwarzschild radius of a  $1 M_{\text{sun}}$  object being 3 km. What is the Schwarzschild radius of a  $10^8 M_{\text{sun}}$  black hole? If you fell through the Schwarzschild radius of a  $10^8 M_{\text{sun}}$  black hole, how many minutes would it take you to fall to the central singularity? The tidal force between your head and your toes at the Schwarzschild radius of a  $1 M_{\text{sun}}$  black hole is about  $10^9 g$ . What is the tidal force at the Schwarzschild radius of a  $10^8 M_{\text{sun}}$  black hole? Would you get ripped apart while entering the Schwarzschild radius of a  $10^8 M_{\text{sun}}$  black hole? Would you get ripped apart somewhere inside the black hole?

- $R_s \sim 3 \times 10^8 \text{ km} - 2 \text{ Mk}$
- Assuming you fall at  $c$  along a degenerate ellipse whose semi-major axis is  $R_s/2$  and using Kepler's 3<sup>rd</sup> law,  $t \sim 1600 \text{ s} - 4 \text{ Mk}$
- $F_t = GM/R_s^3 \sim 10^{-6} g$ ; Human body can withstand force  $\sim 10g$ ; Hence no ripping until,  $R \sim 10^8 m - 4 \text{ Mk}$

2. (a) Are there a greater number of galaxies in the Universe with luminosity higher than  $L^*$  or with luminosity less than  $L^*$ ? (b) Compare the total light in the Universe from relatively bright and relatively dim galaxies. Roughly what is the ratio of the total light emitted by galaxies with  $L > L^*$  to the total light emitted by galaxies with  $L < L^*$ ? (d) The luminosity of the Milky way is about  $2 \times 10^{10} L_{\odot}$  (that's 20 billion times the luminosity of the Sun). How does the Milky Way's luminosity compare to  $L^*$ ? How does the Milky Way's luminosity compare to most of the Galaxies in the Universe? Comment on what this may say about the Cosmological Principle (which may be briefly stated as "we are nowhere special").



- galaxies with  $L < L^*$  are greater in number – 1 Mk
- Look at the dotted curve (Luminosity function). Light from  $L < L^*$  galaxies is almost same as that from  $L > L^*$  galaxies (after eyeballing area under the curve for both sides), although brighter galaxies contribute slightly more due to the bump on the right of  $L^* - 3 \text{ Mk}$
- The Milky Way is a galaxy with  $L \sim L^*$ , which puts it in a select group of the brightest galaxies. It's not a big huge cD galaxy, but it's pretty study as galaxies go. And, indeed, although the Sun as a G-star is pretty wimpy compared to bright hot blue stars, it's pretty study compared to the M-dwarfs that make up the bulk of the stellar population. – 3 Mk
- Since we're in a big galaxy, as galaxies go, and in an above-average mass star, as stars go, does this mean that we're in a special place in the Universe? No. There are lots and lots of other galaxies very much like the Milky Way out there. We know that a good fraction of the light of the Universe comes from the brightest galaxies; where is most of the mass of the Universe? If the "initial mass function" (i.e. how many stars form at different masses) is the same everywhere, then there are about as many stars in giant galaxies as in dwarf galaxies, so it's no real surprise we're in a giant galaxy. Similarly, there are lots and lots of stars like the Sun. There are reasons why it may be easier for live to evolve around a star like the Sun, as compared to an M-dwarf. (A larger range of distances from the star can support liquid water.) – 3 Mk

3. Two stars are observed near to each other on the sky. One star's spectrum indicates that it's a main

sequence O5 star; the other's spectrum indicates that it's a main sequence K2 star. The two stars have the same bolometric magnitude. Are the two stars at the same distance from the sun, or is one closer? If one is closer, which one; and what is the ratio of the distances? You perform a survey of stars in a region of the sky that counts all stars down to visual magnitude 18. You want to know the relative frequency of main-sequence stars of spectral type A and G. In your survey, you find the same number of main-sequence A stars as you do main-sequence G stars. (a) What, qualitatively, can you conclude about the relative number of main-sequence A and G stars in the region of the galaxy you were looking at? (b) What, quantitatively, can you conclude about the relative number of main-sequence A and G stars in the region of the galaxy you were looking at?

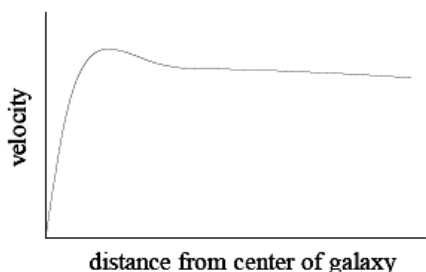
a) Since both appear to be of same magnitude, the Flux-ratio is 1; therefore

$$\frac{F_{O5}}{F_{K2}} = 1 = \frac{L_{O5}}{L_{K2}} \left( \frac{d_{K2}}{d_{O5}} \right)^2$$

- 2 Mk

- b) O5 stars are  $10^6$  times brighter than K2 stars (by obs or by LMR relations applied to Pogson's formula), O5 is 1000 times farther than K2 – 3 Mk
- c) A is brighter than G, hence at the same limiting magnitude ( $18^m$ ) than hence you see more G stars per unit volume than A stars – 2 Mk
- d) Since A stars are 3.5 times brighter, hence  $d_A / d_G = 5$ , hence you are sampling a volume 125 times bigger for A stars than for G stars, so there are 125 times as many G stars as there are A stars – 3 Mk

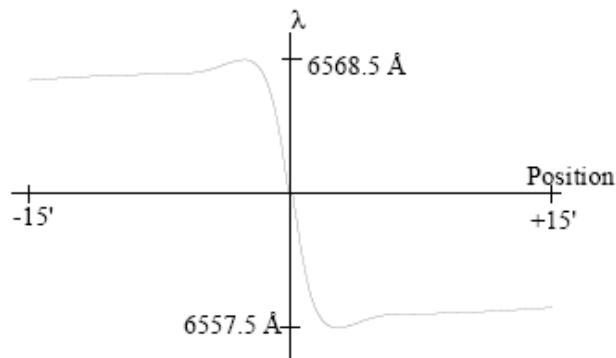
4.



You observe a rotating spiral galaxy edge on: (a) Suppose this galaxy has a rotation curve like a merry-go-round. Make a plot of the observed wavelength of the  $H\alpha$  line vs. observed angular distance from the center of the galaxy. Assume that the galaxy subtends  $15'$  on the sky, and that the maximum velocity you observe is 250 km. (b) Galaxies don't really rotate like merry-go-rounds; rather, they have a rotation curve, which tells you the linear (not angular) velocity of a star or gas cloud at a given distance from the center of the galaxy, that looks something like

shown in the graph on the left. Sketch a plot that roughly shows the observed wavelength of an  $H\alpha$  line vs. observed angular distance from the center of the galaxy, again assuming that the galaxy subtends  $15'$  on the sky and that the maximum velocity you observe is 250 km/s.

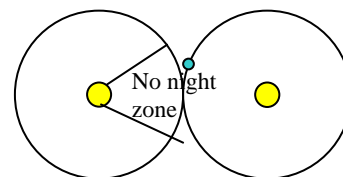
- a) Assuming solid structure, linear velocity varies linearly with distance from center; so does  $H\alpha \sim v(\lambda/c)$  – 3 Mk
- b) End points of the graph would be the maximum  $\Delta\lambda$  on one axis  $\sim 5.5^\circ\text{A}$  &  $15'$  on the other; graph of  $H\alpha$  – 3 Mk
- c) If galaxies are not solid structures then graph is as shown on right – 4 Mk



5.

A planet revolves around a double star system (both companions have equal mass) with a period P. It has an orbit which is approximately circular around both stars and the orbit passes through the first Lagrangian point of the binary system. Will the people residing on the planet (any part of it) ever see any other stars. For how long and how often? (discuss) Give an expression for the longest period of time (in terms of the angular separation of the two stars for a planet-based observer) for which the planet will never experience nightfall. You may take the rotation period of the planet to be T.

- a) diagram – 2 Mk
- b) There will be nightfall for at least some time every day when the angular separation ( $\alpha$ ) of the Sun's is more than  $180^\circ + 2x$ , where  $x$  is the twilight zone  $\sim 6^\circ$  on either side (civil twilight) 2Mk
- c) Longest day will be between positions of  $178^\circ < \alpha < 192^\circ$  – 3 Mk
- d) Planet travels  $12^\circ$  in this phase;  $P = 2\pi/\omega$  gives  $t \sim P/15$  – 2 Mk
- e) In terms of planet-days  $\sim P/(15.T)$  – 1 Mk



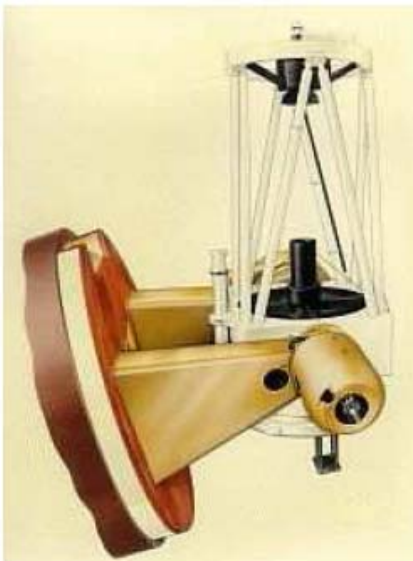


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**Theory Test – II (Time: 2<sup>h</sup>; Mk: 40)**

- The star Vega is at right ascension,  $18^h 37^m$  & declination  $38^\circ 47'$ . Egham (Briton) is at latitude  $51^\circ 26'$  N and longitude  $0^\circ 34'$  W. On 6 October 2005 at 11:00 pm, I find the Greenwich sidereal Time GST = 23h 2m. For an observer in Egham, find the following:
  - What is the hour angle of Vega at this time?
  - What are the altitude and azimuth of Vega at this time?
  - At what sidereal time is (or was) Vega's upper transit?
  - At what time BST (British Standard Time) is its upper limit?
    - 2 Mk, 4 Mk, 2 Mk, 2 Mk

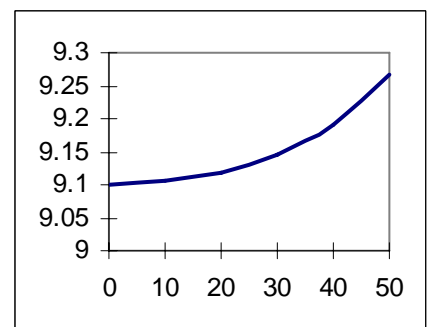


- The figure shows the equatorially mounted Isaac Newton Telescope.
  - By looking at the drawing, estimate roughly the latitude where the telescope is located. (It is in the northern hemisphere)
  - Estimate roughly the hour angle and the declination of the telescope as it is shown.
  - The star Atria has RA  $16^h 49^m$  and Dec  $-69^\circ$ . Is it possible to see Atria through the Isaac Newton Telescope? Explain.
    - The plane of the base plate of the mount making an angle  $\sim 70^\circ$  with the vertical indicates latitude as much. – 2 Mk
    - Assuming the photo is taken perfectly sideways, HA  $\sim -1h$ ; assuming the telescope to be looking at zenith,  $\delta \sim 70^\circ$  – 2 Mk
    - A southern dec of  $69^\circ$  can never be seen from the telescope due to the star being south circumpolar from it – 1 Mk

- A star is observed at different values of the Zenith distance  $z$  to have the visual magnitudes shown in table. Extrapolate the data to zero air mass to determine the magnitude of the star above the atmosphere. Another star is observed at an altitude of  $45^\circ$  with a visual magnitude as measured on the ground of 8.5. What is the magnitude above the atmosphere?

zenith distance	$m_V$
$10^\circ$	9.105
$20^\circ$	9.119
$30^\circ$	9.146
$40^\circ$	9.192
$50^\circ$	9.267

- Plot a graph, and extrapolate the parabola to touch  $\sim 9.1^m$  –
- 



- An astronomer observes three blobs of gas in the plane of the Galaxy. Each blob of gas is at a measured galactic longitude  $l$  and has a measured radial velocity  $V_r$  in the Local Standard of Rest (where a positive velocity means it's moving away from us, and a negative velocity means it's moving toward us). He measures for each object: Derive an expression for  $R$ , the distance of a blob from the center of the Galaxy, in terms of known quantities. Known quantities include  $R_0$ , the distance of the
 

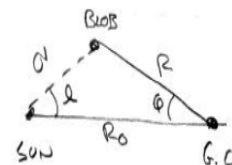
Blob	$l$	$V_r$ (km/s)
A	$23.8^\circ$	89
B	$-26.6^\circ$	-98
C	$111.7^\circ$	-68

Sun from the center of the Galaxy;  $V = V_0$ , the velocity of an object relative to the Galaxy (assumed to be the same everywhere in the galactic disk); and the measured quantities  $l$  and  $V_r$ . (b) Derive an expression for  $d$ , the distance from the Sun to the observed blob of gas, in terms of the known quantities from (a) and  $R$ . (c) For the three blobs given, evaluate and tabulate all possible values of  $R$  and  $d$ . Use  $R_0 = 8$  kpc and  $V = V_0 = 220$  km/s. (d) Sketch a schematic diagram of the Galaxy, as seen by an observer looking down on the plane of the Galaxy. Indicate the position of the Galactic Center, the Sun, and the possible positions of each of the three blobs on this diagram.

a.  $\omega$  w.r.t. observer = actual  $\omega$  -  $\omega$  w.r.t. galactic center gives,  $R = \frac{VR_0 \sin l}{V_r + V_0 \sin l}$  - 4Mk

b.  $\frac{\sin \phi}{d} = \frac{\sin l}{R}$  Law of cosines gives,  $d^2 = (2R_0 d \cos l) d - (R_0^2 - R^2)$ ,

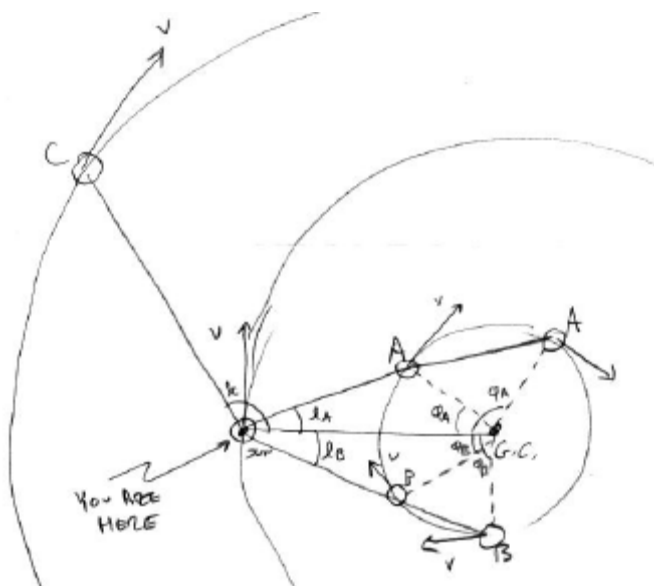
giving  $d = R_0 \cos l \pm \sqrt{R^2 - R_0^2 \sin^2 l}$  - 4Mk



Blob	R Kpc	D Kpc	Q for plotting
A	4	4.96 or 9.68	$30^\circ$ or $102^\circ$
B	4	5.37 or 8.93	$-37^\circ$ or $-90^\circ$
C	12	6.46	$30^\circ$

- c. accurate table - Mk
- d. 3Mk

4



# Practical Problems

1. The response of a one-dimensional 16-pixel CCD to a point source of flux 1 Jansky is as below, when the point source is located directly “above” pixel 7:

Count:	0	0	0	0	1	2	4	5	4	2	1	0	0	0	0	
Pixel:	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

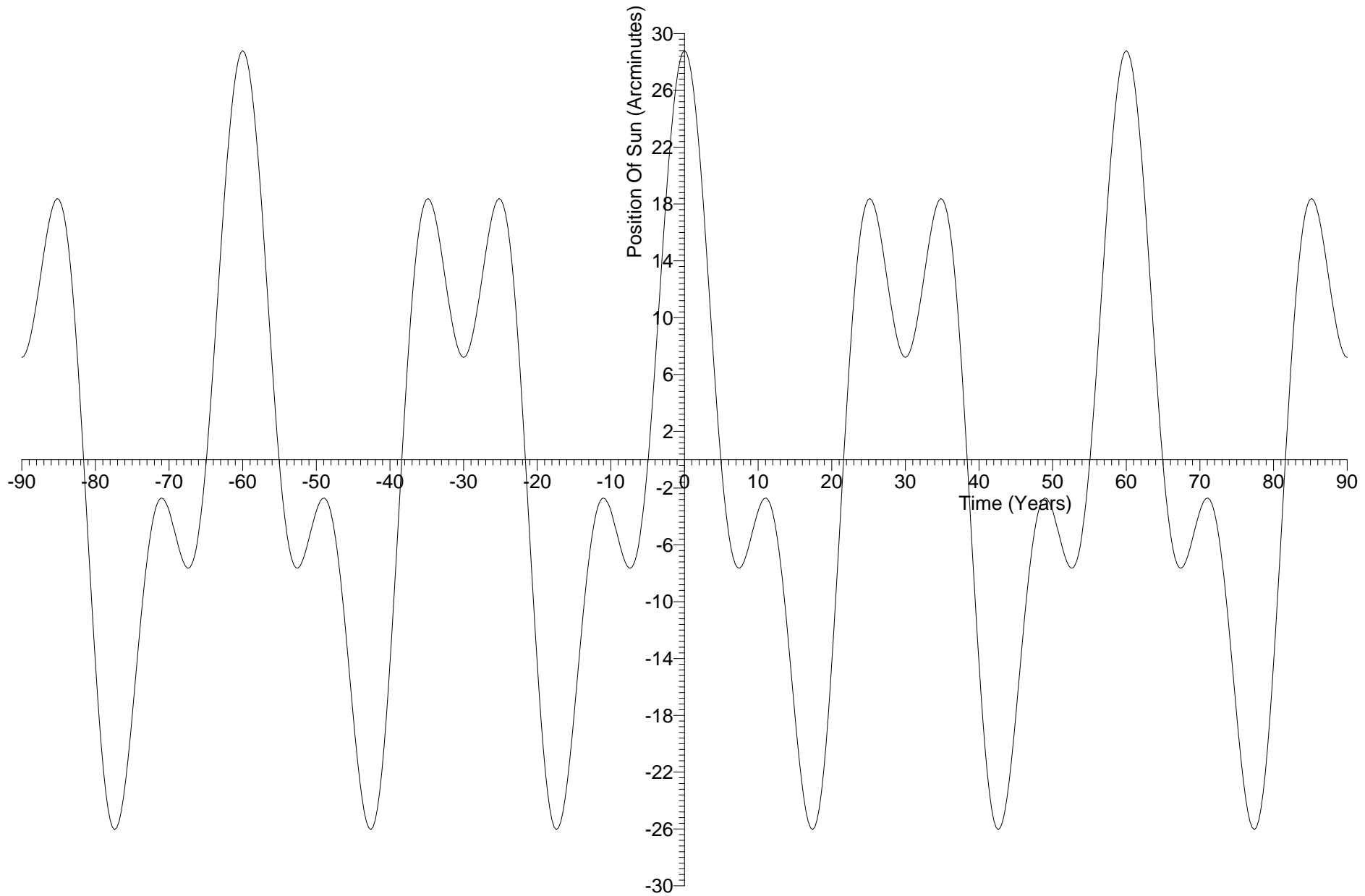
Also, 1 pixel = 1” in the sky.

A CCD image taken of a 1D area of the sky shows the following counts in the CCD:

Count:	3	6	12	15	12	6	8	10	20	25	22	14	13	10	8	4
Pixel:	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Prepare a map of the area of the sky.

2. The graph on the next page shows the plot of change of the position of the sun as seen from Earth due to Jupiter and Saturn. Find time periods and masses of the two planets.



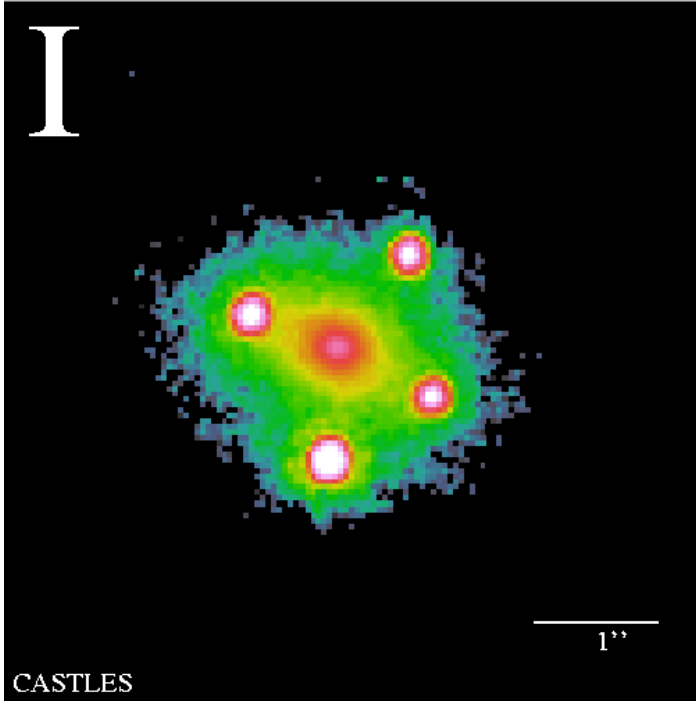


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**Lab Test II**

1.



A gravitational lens is a massive galaxy (star in case of micro-lensing) with a super-massive black-hole at its core. This lens deflects the light from objects behind it (as seen by observer) which creates multiple images of the same object around the lens. These images may have different morphology, but they exhibit the same redshift and spectral index, which helps one identify their source. In case the lens is exactly collinear with the source and the observer, we get many images symmetrically spread out around the lens forming an "Einstein's Ring or Cross". The

radius of the ring is given as 
$$\theta = \sqrt{\frac{4GM D_{ls}}{c^2 D_l D_s}}$$
,

where M is the mass of the lensing galaxy,  $D_l$  is its distance from the observer,  $D_s$  is the distance of the source from the observer and  $D_{ls}$  is the physical separation of the source from the lens.

One such Cross (Q2237+030) as observed by the Harvard CASTLES survey is shown in the picture. Angular separations between the four images (B, C, D & G) from the lens (A) are given below. The redshift of the lens and the source are 0.04 & 1.69, respectively. The magnitudes in the I-band (9000 Å) for the lens and the source are 14.15 & 15.16 (collective of all images), respectively.

Observations	A	B	C	D	G
Position					
RA (")	0	-0.673±0.003	0.635±0.003	-0.866±0.003	-0.075±0.004
Dec (")	0	1.697±0.003	1.210±0.003	0.528±0.003	0.939±0.003

- Bearing in mind that all cosmological distances are scaled by the Hubble parameter (constant if you like), and assuming the lens to be a typical galaxy, estimate the Hubble parameter in the vicinity of the lens.
- Hence estimate the Hubble-corrected distances of the lens and the source.
- Hence estimate the Luminosity of the source.

2.

Sp (I)	$U-V$		$B-V$	$V-R$	$V-I$	$V-J$	$V-K$	$V-L$	$V-M$	$V-N$	B.C.
	Ia	Ib									
O9	-1.46	-1.46	-0.31	-0.15	-0.47	-0.73	-0.94	-1.00	—	—	—
O9.5	-1.44	-1.44	-0.30	-0.13	-0.45	-0.70	-0.90	-0.96	—	—	—
B0	-1.38	-1.37	-0.27	-0.11	-0.38	-0.62	-0.79	-0.84	—	—	-2.29
B0.5	-1.32	-1.30	-0.25	-0.09	-0.32	-0.54	-0.70	-0.74	—	—	-1.99
B1	-1.26	-1.24	-0.22	-0.08	-0.27	-0.47	-0.60	-0.62	—	—	-1.82
B2	-1.15	-1.10	-0.18	-0.05	-0.20	-0.36	-0.45	-0.46	—	—	-1.53
B3	-1.03	-0.98	-0.14	-0.02	-0.14	-0.29	-0.36	-0.36	—	—	-1.27
B5	-0.86	-0.78	-0.10	+0.02	-0.05	-0.20	-0.22	-0.20	—	—	-0.91
B6	-0.80	-0.74	-0.08	+0.02	-0.03	-0.16	-0.16	-0.13	—	—	-0.85
B7	-0.73	-0.67	-0.06	+0.02	-0.01	-0.12	-0.10	-0.06	-0.10	-0.10	-0.75
B8	-0.64	-0.57	-0.03	+0.02	+0.02	-0.08	-0.03	+0.01	-0.03	-0.03	-0.64
B9	-0.58	-0.50	-0.01	+0.02	+0.05	-0.03	+0.03	+0.08	+0.03	+0.03	-0.52
A0	-0.47	-0.40	+0.01	+0.03	+0.08	+0.01	+0.10	+0.15	+0.10	+0.10	-0.38
A1	-0.35	-0.29	+0.03	+0.05	+0.11	+0.06	+0.16	+0.21	+0.16	+0.16	-0.30
A2	-0.22	—	+0.05	+0.07	+0.14	+0.10	+0.22	+0.27	+0.22	+0.22	-0.21

Given above is a table of color-difference magnitudes measured for stars classified according to their spectra. The bolometric correction to be applied for the star magnitudes is also specified. Given below is the calibration chart for the various bands that were used to measure the color-differences of the stars. Use this data to determine the difference in temperatures of the B7 & B8 classes of stars.

Filter band	$\lambda_0$	Absolute flux density for mag = 0.00	
		$F_\lambda$	$F_\nu$
<i>U</i>	0.36 $\mu$	$4.35 \times 10^{-12}$ W/cm <sup>2</sup> $\mu$	$1.88 \times 10^{-23}$ W/m <sup>2</sup> Hz
<i>B</i>	0.44 $\mu$	$7.20 \times 10^{-12}$ W/cm <sup>2</sup> $\mu$	$4.44 \times 10^{-23}$ W/m <sup>2</sup> Hz
<i>V</i>	0.55 $\mu$	$3.92 \times 10^{-12}$ W/cm <sup>2</sup> $\mu$	$3.81 \times 10^{-23}$ W/m <sup>2</sup> Hz
<i>R</i>	0.70 $\mu$	$1.76 \times 10^{-12}$ W/cm <sup>2</sup> $\mu$	$3.01 \times 10^{-23}$ W/m <sup>2</sup> Hz
<i>I</i>	0.90 $\mu$	$8.3 \times 10^{-13}$ W/cm <sup>2</sup> $\mu$	$2.43 \times 10^{-23}$ W/m <sup>2</sup> Hz
<i>J</i>	1.25 $\mu$	$3.4 \times 10^{-13}$ W/cm <sup>2</sup> $\mu$	$1.77 \times 10^{-23}$ W/m <sup>2</sup> Hz
<i>K</i>	2.2 $\mu$	$3.9 \times 10^{-14}$ W/cm <sup>2</sup> $\mu$	$6.3 \times 10^{-24}$ W/m <sup>2</sup> Hz
<i>L</i>	3.4 $\mu$	$8.1 \times 10^{-15}$ W/cm <sup>2</sup> $\mu$	$3.1 \times 10^{-24}$ W/m <sup>2</sup> Hz
<i>M</i>	5.0 $\mu$	$2.2 \times 10^{-15}$ W/cm <sup>2</sup> $\mu$	$1.8 \times 10^{-24}$ W/m <sup>2</sup> Hz
<i>N</i>	10.2 $\mu$	$1.23 \times 10^{-16}$ W/cm <sup>2</sup> $\mu$	$4.3 \times 10^{-25}$ W/m <sup>2</sup> Hz

3. The following tables give data about 4 planets around a star HD75732. The planet data table lists the mass in terms of Jupiter, semi-major axis, eccentricity, orbital period, average radius of orbit and Irradiance of the planet (average energy reaching the planet's surface assuming zero albedo).

Irradiance is given by  $I = \frac{L_*}{4\pi a^2 \left(1 + e^2/2\right)^2}$  where  $L_*$  is the star's luminosity and the planet's

semi-major axis.

- a. If the planet is within the star synchronous orbit, i.e. where it can create significant tidal pull on the star causing it to bulge, and if it is going too fast or too slow the bulge can absorb the angular momentum of the planet and cause it to spiral inwards and fall into the star. Hence for stability the planet must be out of the star synchronous orbit, given by

$$r_s = \left( \frac{P_{rot}^2 GM_*}{4\pi^2} \right)^{1/3}$$

. Which of the planets are thus unstable?

- b. Estimate the sizes of each of the planets (approximate for gaseous or solid surfaces cautiously). Are any of them as big as the earth? If not, how big a telescope would be required to see an Earth-sized planet and how far away should the planet be for adequate irradiance?
- c. Which of the planet is closest to the Earth in terms of possibility of life? Elaborate

**Host-Star Data**

Identifier	Spectral Type	Mass	Luminosity	$T_{eff}$	$P_{rot}$	$R_*$	Age
		$M_\odot$	$L_\odot$	$K$	Days	$R_\odot$	Gyr
HD75732	G8V	0.95	0.61	5250	38.5	0.96	5.0

## Planet Data

Identifier	Mass M <sub>J</sub>	a AU	e	P <sub>orb</sub> Days	r <sub>av</sub> UA	I 10 <sup>6</sup> erg/cm <sup>2</sup> s
HD75732e	0.045	0.038	0.174	2.81	0.0386	557.8
HD75732b	0.84	0.115	0.02	14.65	0.115	62.7
HD75732c	0.21	0.241	0.339	44.276	0.2548	12.78
HD75732d	4.05	5.9	0.16	5360.	5.9752	0.023