United States Physics Team

Entia non multiplicanda sunt praeter necessitatem

1997 Creative Response Portion of Exam 1

4 Questions, 60 Minutes

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

Show all work, as partial credit may be earned.

Communicate! The grader will not attempt to read your mind.

A hand-held calculator may be used. Its memory must be cleared of data and programs.
Calculators may not be shared.

Possibly useful approximations:

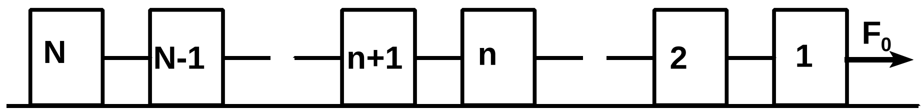
$(1 + x)^n \approx 1 + nx$ for $|x| << 1$
$\cos\theta \approx 1 - \theta^2/2!$ for $\theta << 1$
$\sin\theta \approx \theta$ for $\theta << 1$

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

1. A collection of N identical blocks, each of mass m, are connected by unstretchable ropes of negligible mass. The blocks are on a horizontal surface. An external force F_o acts on Block 1, pulling horizontally to the right. What is the tension in the rope connecting Block n to Block $n+1$, if:

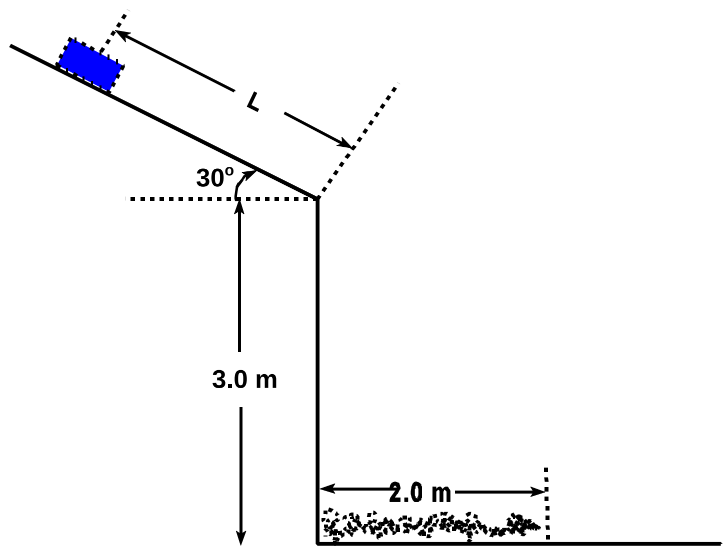
(a, 8) There is no friction between the blocks and the surface?

(b,12) The coefficient of kinetic friction between each block and the surface is μ_k? (F_o is large enough to accelerate the blocks despite the friction.)

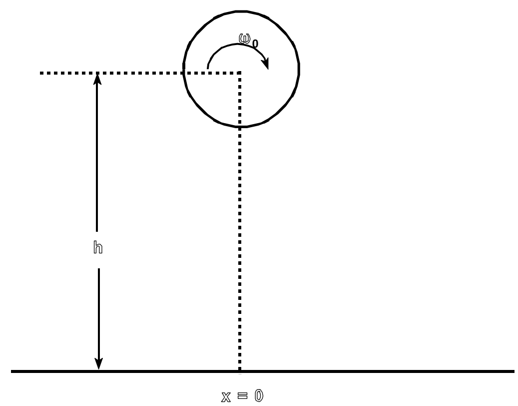


2. **(a, 10)** A box of nails begins sliding from rest on the roof a house. The roof makes an angle of 30° above the horizontal. The coefficient of sliding friction is $\mu_k = 1/4$. The box slides off the edge of the roof with a speed of 3.5 m/s. What is the distance L the box slides on the roof before falling off? Use g = 9.8 m/s^2 (which also = 9.8 N/kg).

(b, 10) The flower bed extends from the side of the house and is 2.0 m wide. The edge of the roof is 3.0 m above the ground. Does the box of nails land in the flowers? Justify your answer with a calculation that shows where the box lands. Neglect air resistance.



3. A basketball of mass *M*, radius *R*, and moment of inertia *I* about its center of mass (CM) is set spinning with angular velocity ω_o about a horizontal axis through its CM. The CM is originally at rest and located at the height *h* above the floor. The basketball is dropped while spinning, and subsequently collides with the floor. Neglect air resistance.



(a, 3) Let K_1 be the basketball's kinetic energy just before it collides with the floor. Write K_1 in terms of the given parameters and any needed constants.

(b, 17) Immediately after the first bounce, the basketball is no longer spinning, and its kinetic energy is βK_1, where $\beta < 1$ is a known factor. What are the horizontal and vertical components of the basketball's velocity immediately after the first bounce?

4. Consider the gravitational force F on a planet due to the Sun when that force includes a small perturbation Γ giving a departure from Newton's law of gravitation,

$$F = (1 + \Gamma)(GMm/R^2)$$

where M is the Sun's mass, m is the planet's mass, G is Newton's constant, R is the distance between the center of the Sun and the center of the planet, and we take Γ to be a constant $<< 1$. (For example, general relativity provides such a perturbation.) Approximate the orbit as circular.

(a, 7) Show that the planet's period is $T \approx T_o(1 - \frac{1}{2}\Gamma)$, where T_o would be the planet's period if there were no perturbation.

(b, 7) In one revolution, a planet moving under the purely Newtonian force would travel through the angle 2π. But with the perturbation, in the same amount of time the planet travels through an additional angle δ. Calculate δ in terms of Γ.

(c, 2) The general theory of relativity contributes the perturbation $\Gamma = 6v^2/c^2$ to Newton's law of gravitation, where v is the speed of the planet relative to the Sun, and c is the speed of light. The precession angle δ of part (b) may be approximated as $\delta \approx (6\pi/c^2)(GM/R)$. Demonstrate this claim that $\delta \approx (6\pi/c^2)(GM/R)$, by using your result of part (b) and the given perturbation.

(d, 4) Using the result $\delta \approx (6\pi/c^2)(GM/R)$, and using the data below, calculate the numerical value of δ for the planet Mercury. Express your answer in seconds of arc per century. This is our simple model's prediction of general relativity's contribution to the precession of Mercury's orbit.

Data: Period of Mercury's orbit = 88 days
Radius of Mercury's orbit = 5.8×10^{10} m
$G = 6.7\times10^{-11}$ Nm^2/kg^2
Mass of Sun = 2.0×10^{30} kg
$c = 3.0\times10^8$ m/s
3600" = 1 degree.

[This problem adapted from A.P. French, *Newtonian Mechanics*, Norton & Co., NY, 1971.]

United States Physics Team

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1997 Exam 2

Instructions:

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

1. **Please show all work, and do not hesitate to use explanatory words in addition to equations. Partial credit will be awarded**.

2. Begin each problem on a fresh sheet of paper. In the upper right-hand corner of each page, write your name, problem number, and the page number/total number of pages for this problem (e.g., "A2, 3/4" for "Problem A2, page 3 of four").

3. You may use a non-programmable calculator, but may not use any tables, books, notes, or collections of formulas.

4. Work Part A first. You have ninety minutes to complete all four problems.

5. After you have completed Part A, you may take a break.

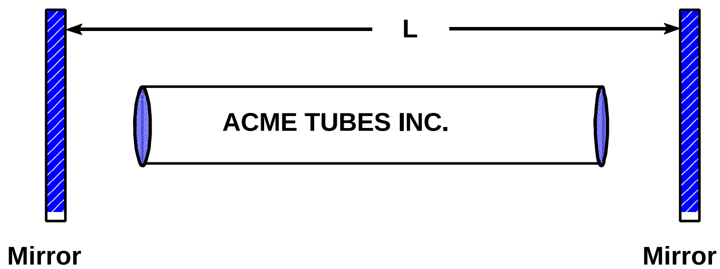
6. Then work Part B. You have 90 minutes to complete both problems.

Possibly useful information:

Gravitational field on Earth's surface: $g = 9.8$ N/kg
Gravitational constant: $G = 6.67\times10^{-11}$ N-m^2 / kg^2
Coulomb constant: $k = 1 / 4\pi\epsilon_o = 8.99\times10^9$ N-m^2 / C^2
Speed of light in vacuum: $c = 3.00\times10^8$ m/s
Permeability of empty space: μ_o, where $\mu_o\epsilon_o = 1/c^2$
Ideal gas constant $R = 8.31$ J / mol-K
Boltzmann's constant: $k_B = 1.38\times10^{-23}$ J/K
Avagadro's number: $N_A = 6.02\times10^{23}$
1 electron volt = 1 eV = 1.60×10^{-19} J
Planck's constant: $h = 6.63\times10^{-34}$ J-s = 4.14×10^{-15} eV-s

$(1 - x)^{-1} = 1 + x + x^2 + x^3 + \ldots$ for $|x| < 1$
$(1 + x)^n = 1 + nx + (1/2!)n(n-1)x^2 + \ldots$ for $|x| < 1$
$\cos\theta \approx 1 - \theta^2/2$ for $\theta << 1$
$\sin\theta \approx \theta$ for $\theta << 1$

A1. A laser may be constructed from two main components: a resonant cavity of length L formed from two highly reflecting flat mirrors, and a gas-filled tube that emits light when excited by a high voltage. Placed inside the otherwise evacuated cavity, the tube creates light which bounces back and forth between the mirrors such that standing light waves are produced within the cavity. (The radiation can be let out through a tiny window in one of the mirrors, but in this problem we are interested in what goes inside the cavity.)



(a, 5) What are the frequencies possible for the standing light waves in the cavity? Express your answer in terms of the cavity length L and the speed of light in vacuum, c.

(b, 5) Suppose the plasma tube emits light of a pure frequency $f_o = 5\times10^{14}$ Hz. Which standing wave mode (specified by an integer n_o) is excited in the cavity if $L = 1.5$ m?

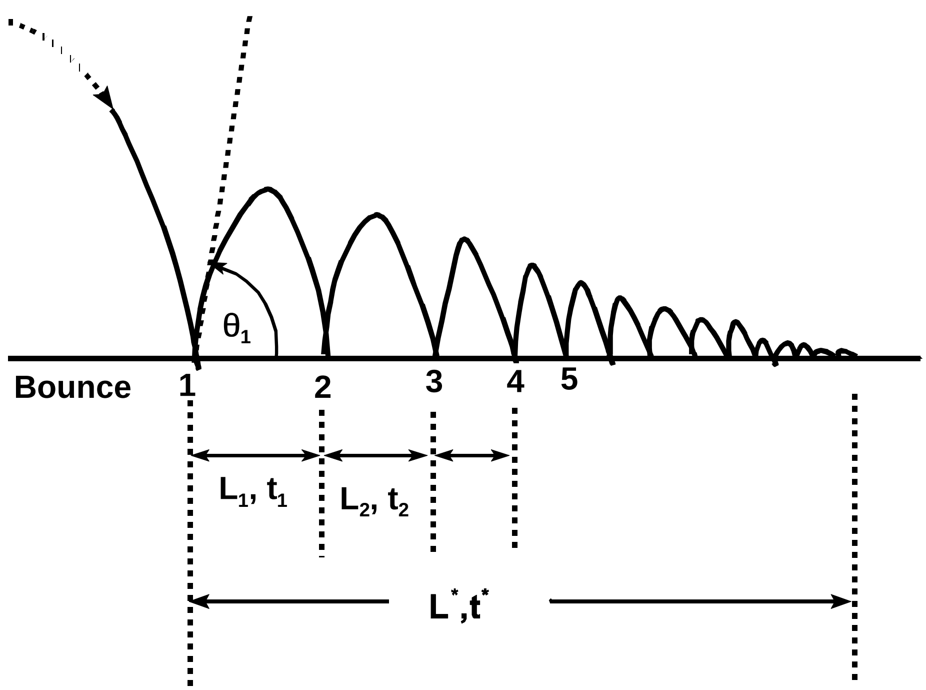
(c, 5) Suppose the plasma tube emits light not of a pure frequency, but emits light having all the frequencies in the range $f_o \pm \Delta f$, where $f_o = 5\times10^{14}$ Hz, and $\Delta f = 1\times10^{9}$ Hz. How many standing wave modes are excited in the cavity if $L = 1.5$ m?

(d, 5) Using the f_o and Δf of part (c), what is the largest value of L such that only *one* standing wave mode will be excited in the cavity, thereby giving the laser only *one* output frequency?

(Adapted from A.P. French, *Vibrations and Waves*, Norton, NY, 1971).

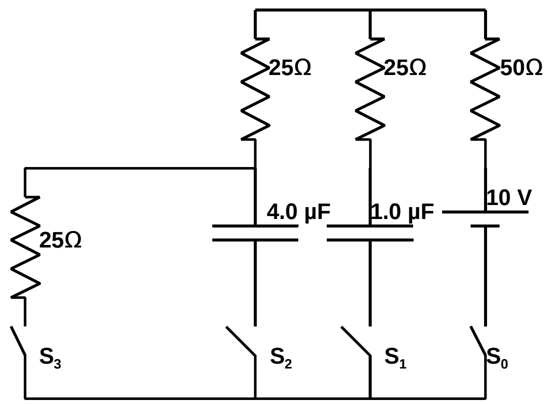
A2. **(20)** A ball is tossed onto the floor, where it makes a succession of bounces as illustrated in the figure below. Assume that because of internal elasticity and friction with the floor, at each bounce the magnitude of the vertical velocity component is reduced by a factor ϵ_y and the horizontal component is reduced by a factor ϵ_x. That is, if $v_{oy,n+1}$ denotes the *y*-component of the velocity as the ball emerges from the (*n+1*)st bounce, then $v_{oy,n+1} = \epsilon_y v_{oy,n}$, and similarly for the *x*-component. Note that ϵ_y and ϵ_x are < 1. Thus after each bounce, the ball moves slower and hops a shorter distance than it did after the preceding bounce.

Let the ball's succession of bounces traverse a total horizontal distance L^* which takes the time t^*. (As a practical matter, we measure L^* and t^* as the length and time where the bounces become imperceptible; mathematically, the number of bounces goes to infinity.) Find θ_1, the angle the ball's velocity makes with the horizontal immediately after the first bounce, written in terms of L^*, t^*, ϵ_y, ϵ_x, and needed constants. Neglect air resistance.



A3. The circuit shown consists of:

a 10 volt battery,
a 1.0 μF capacitor,
a 4.0 μF capacitor,
a 50 Ω resistor,
three 25 Ω resistors, and four switches, labeled S_o, S_1, S_2, S_3 . All switches are initially open. In all cases assume that the circuit is completely insulated from its surroundings.



(a, 5) Switches S_o, S_1, S_2 are closed. Switch S_3 remains open. After a very long time, what is the charge on each capacitor?

(b, 5) Switch S_3 is also closed, so that all four switches are closed. After a very long time, what is the charge on each capacitor?

(c, 5) Switches S_o and S_3 are opened simultaneously. Switches S_1 and S_2 are left closed. After a very long time, what is the charge on each capacitor?

(d, 5) Switches S_2 and S_3 are opened. Switches S_o and S_1 are closed. After a very long time has passed you insert into the 1.0 μF capacitor a slab of dielectric material. After the dielectric is inserted, it totally fills the region between the plates. The material's dielectric constant is 3.0. What is the total work done in inserting the dielectric (that it, what is the net work done by you *and* the battery)?

A4. An atom in an excited state typically decays by emitting a photon. The atom is originally in some excited state S', and after the decay it is in the ground state S. We usually think of the photon as carrying away the energy difference between states S' and S. In this problem we examine this process closely.

(a, 4) Consider a stationary, isolated atom in some excited state S'. Before decay the mass of the atom is M'. The atom decays into the ground state S. In the ground state the mass of the atom is M. Thus the energy $(M'-M)c^2$, which we denote here as E_o , is the energy that is released by the atom, and is therefore an upper limit on the energy that the photon could possibly carry away (c is the speed of light). Let E denote the energy of the outgoing photon. Without solving any equation, by merely giving an qualitative argument, explain why E must be strictly less than E_o in the case of an isolated excited atom undergoing radiative decay.

(b, 6) Quantify your answer to part (a) by showing that, approximately,

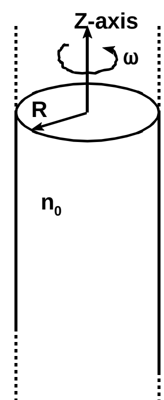
$$E \approx E_o [\, 1 - (E_o / 2Mc^2) \,], \text{ where } E_o / 2Mc^2 << 1.$$

(c, 5) Consider an atom decaying from the excited state S' to the ground state S, where $Mc^2 = 2\times10^{11}$ eV and $E_o = 4\times10^5$ eV. A photon energy corresponds to a radiation frequency. Thus the energy difference $E_o - E$ corresponds to a frequency shift $\Delta f = f_o - f$. Calculate the fractional frequency shift $\Delta f / f_o$.

(d, 5) Another way to produce a frequency shift is the Doppler effect, produced by relative motion between source and observer. The atom of part (c) emits the photon energy E, which corresponds to light frequency f. With what speed and in what direction would an observer have to move relative to this photon, to Doppler-shift its frequency back to the value f_o corresponding to the energy E_o ? The observer's speed v is $<< c$.

B1. Consider an infinitely long, cylindrical cloud of electrons. The cloud has radius R, the number density of electrons has the uniform value n_o , the charge of each electron is $-e$ (where $e > 0$), and the mass of each electron is m. The cloud is surrounded by vacuum. There is a constant, uniform magnetic field along the cylinder's axis (the z-axis), so that $\mathbf{B} = B_o\mathbf{k}$, with $B_o > 0$. The cloud of electrons is rotating about the z-axis with angular velocity ω as shown in the figure. The speeds are non-relativistic. [A gas of charged particles is called a "plasma."]

(a, 6) Find the electric field at the radius $r < R$ within the cloud.



(b, 6) Find the total force on an electron located at the radius r within the cloud, neglecting any magnetic field induced by the cloud's rotation. (This neglect will be justified in part (f) below.)

(c, 8) Using Newton's second law, show that there are two values of ω possible for the electrons to steadily orbit the cylinder's axis. Write these values of ω in terms of two other frequencies relevant to such problems: the cyclotron frequency $\omega_c = e B_o/m$, and the "plasma frequency" $\omega_p = [n_o e^2 / \epsilon_o m]^{1/2}$, where ϵ_o is the permittivity of empty space.

(d, 6) Find the largest electron number density $(n_o)_{max}$ that can be confined by the applied magnetic field $B_o\mathbf{k}$. Write your answer in terms of the magnetic field energy density $B^2/ 2\mu_o$ and the electron's mass energy mc^2.

[This theoretical limit is called the *Brillouin limit*, and plays a key role in the confinement of plasmas consisting of particles all of like charge.]

(e, 8) At a point located at radius r within the cylindrical cloud, calculate the magnetic field induced by the cloud's rotation. Assume that the field induced on the axis of symmetry is zero.

(f, 6) Determine the force on a charge in the cloud, due to the induced magnetic field of part (e). Show that the magnitude of this force divided by the magnitude of the electric force is v^2/c^2, where v is the speed of the charge and c is the speed of light. [This shows that the effect of the induced field is ignorable when the rotation speeds are non-relativistic.]

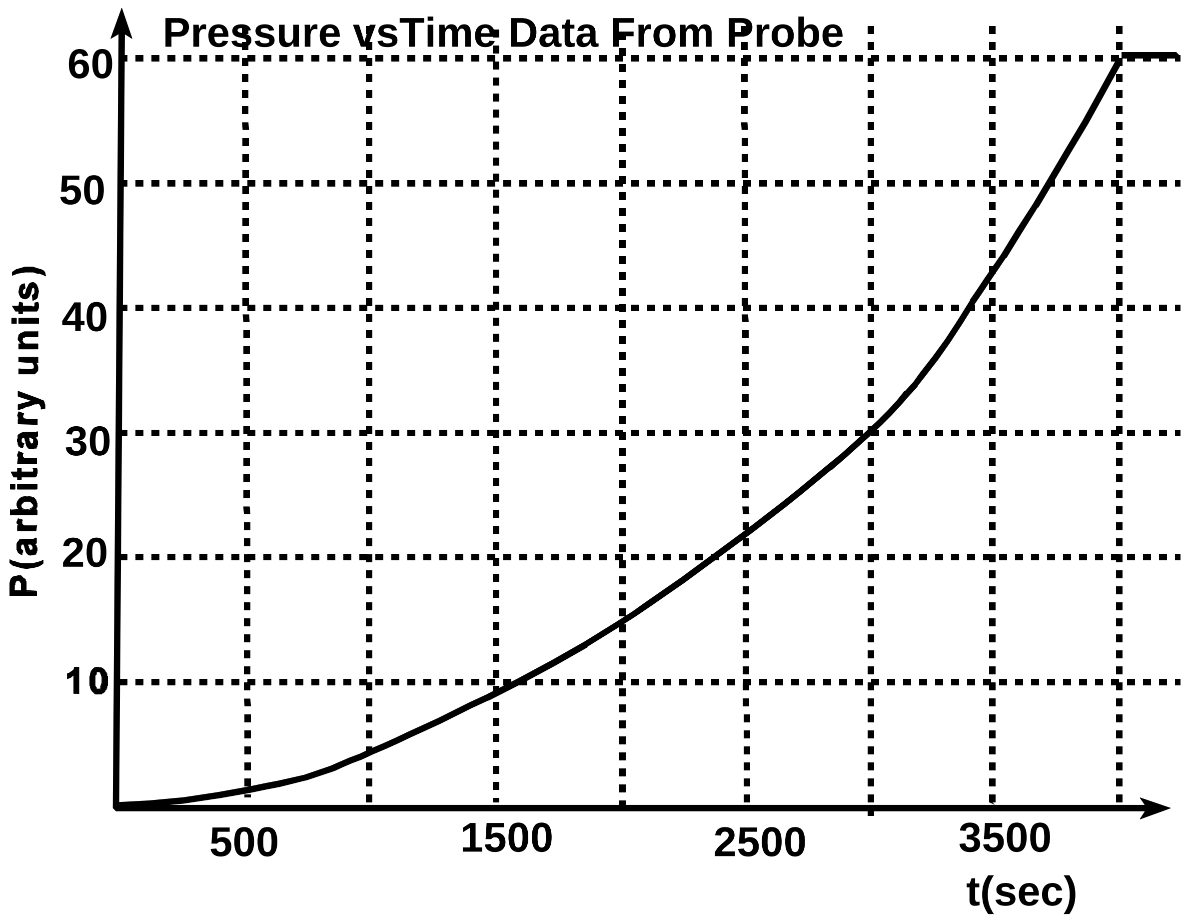
B2. An unmanned space probe approaches the surface of a planet whose atmosphere is pure CO_2 (molecular weight M = 44 gm/mol). Upon entering atmosphere, the probe descends straight down at a constant speed v_o , recording information about the atmospheric pressure, which is shown on the attached graph. Unfortunately, a technician forgot to calibrate the pressure-measuring instrument, so that the P-axis on the pressure vs. time graph has no units! (The technician has been reprimanded, but fortunately, as leader of the data analysis team, you have figured out how to use the data anyway.) Upon reaching the planet's surface, the probe reports the surface temperature to be 400 K, and the gravitational field there to be 9.9 N/kg. The radius of the planet is 5.0×10^6 m. Model the atmosphere locally as an ideal gas.

(a, 5) Apply Newton's Second Law to a small slab of the atmosphere of thickness Δy that is in static equilibrium, to show that the change in pressure ΔP between the top and bottom sides of the slab is given by $\Delta P = \pm\rho g\Delta y$, where ρ the atmosphere's density and g is the local gravitational field (the plus or minus sign is determined by the choice for the positive direction of the vertical y-axis; work out the sign in terms of your choice).

(b, 15) Estimate the velocity v_o with which the probe descends to the surface.

(c, 15) Estimate the temperature of the atmosphere 15 km above the surface. Neglect the variation of the planet's gravitational field between the surface and this height.

(d, 5) Justify the neglect of the variation of the planet's gravitational field between the surface and 15 km above the surface. Suggestion: Calculate the fractional difference between the gravitational field's magnitude on the surface and its value 15 km above the surface, and assess whether that difference is significant compared to the accuracy of the estimate made in part (c).


Pressure vsTime Data From Probe
P(arbitrary units)
60
50
40
30
20
10
500
1500
2500
3500
t(sec)

Preliminary Exam

Open Response Questions

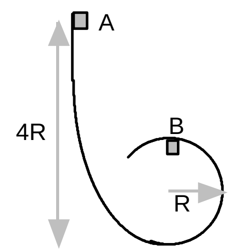
1. A tennis ball launcher is placed on the floor at the front end of a train that has a uniform forward acceleration of 2.00 m/s^2. The launcher projects a ball at an initial speed of 25.0 m/s with respect to the train toward its rear. The ball achieves a maximum height of 10.0 m. Ignore air resistance.

(10) a. Find θ_o, the angle of launch with respect to the floor.

(15) b. Find how far from the front end of the train the ball lands.

(contributed by Leaf Turner)

2. A cube of mass M starts at rest at point A at a height of $4R$, where R is the radius of the circular part of the track. The cube slides down the track and around the loop. The cube is very small compared to the size of the track.



(15) a. Assuming the track is frictionless, find the force the track exerts on the cube at point B. Express your answer as a function of Mg.

(10) b. The cube is replaced by a sphere with mass M and radius r ($r << R$). Assuming that the sphere starts at the same height 4R and rolls without slipping, find the force the track exerts on the sphere at point B. Express your answer as a function of Mg.

(contributed by Mary Mogge)

3. Two masses, m_1 and m_2, attached to equal length massless strings, are hanging side-by-side just in contact with each other. Mass m_1 is swung out to the side to a point having a vertical displacement 0.20 m above mass m_2. It is then released from rest and collides elastically with the stationary hanging mass m_2. Each of the masses is observed to rise to the same height following the collision.

(20) a. Find the numerical value of this height.

(5) b. The masses swing back down and undergo a second elastic collision. Describe what happens following the second collision. (A proof is not required.)

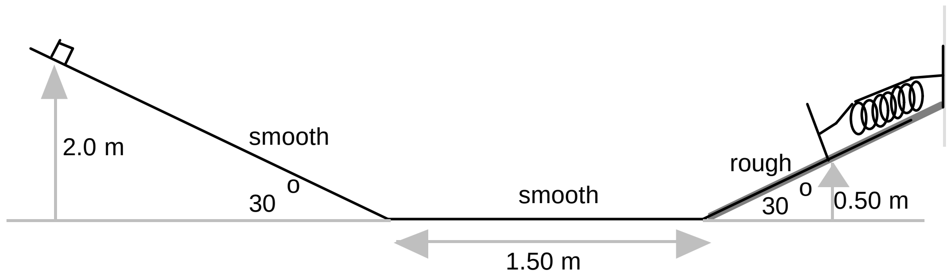
(contributed by Leaf Turner)

(25) 4. A car accelerates uniformly from rest. Initially, its door is slightly ajar. Calculate how far the car travels before the door slams shut. Assume the door has a frictionless hinge, a uniform mass distribution, and a length L from front to back.

(contributed by Leaf Turner)

The 1999 Preliminary Examination open response questions were written by the coaches of the United States Physics Team. The coaches are: Academic Director, Dr. Mary Mogge – Professor of Physics at California State Polytechnic University, Pomona, CA; Senior Coach, Dr. Leaf Turner – Physicist in the Theoretical Division of Los Alamos National Laboratory, Los Alamos, NM; Dr. Warren Turner – Physics Teacher at St. Paul's School, Concord, NH. The coaches would also like to thank former academic director, Dr. Larry Kirkpatrick for his many helpful comments.

1999 Semi-Final Exam
Part A



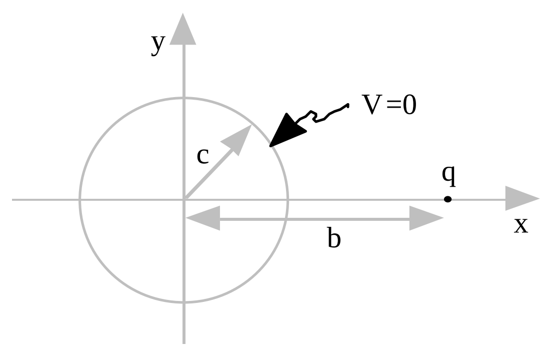
A1. A small crate of weight 5.0 N is released from rest at a height 2.0 m up a smooth inclined plane. The crate slides down the plane and across a smooth 1.50-m floor to a rough plane where a spring is located. The bottom of the spring is located at a height of 0.50 m. The spring constant is 20 N/m. The coefficient of kinetic friction between the rough plane and crate is $1/\sqrt{3}$ and the coefficient of static friction is $1/\sqrt{2}$.

a. (10) What is the maximum height the crate reaches on the right-hand inclined plane?

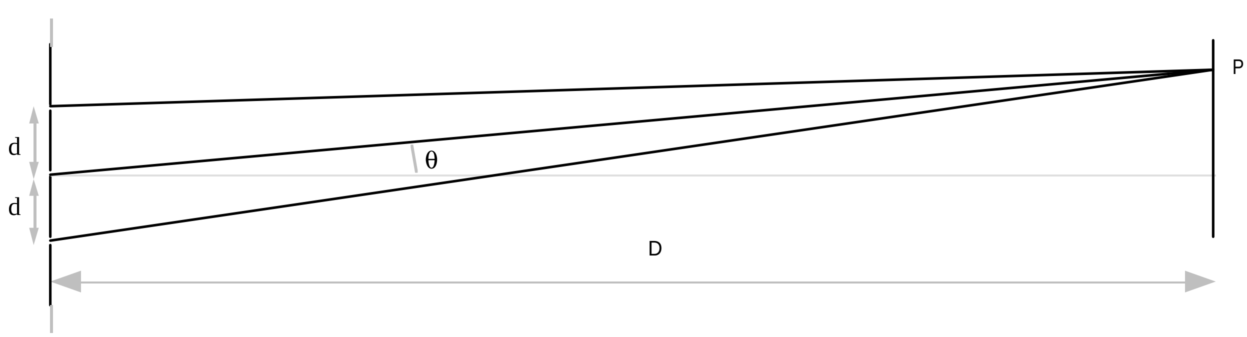
b. (10) Where does the crate permanently come to rest? How many times does the crate travel up the rough plane?

(contributed by Mary Mogge)

A2. (20) Assume the electric potential to be zero at infinity. A charge q is located on the x-axis at $x = b$. One additional charge is placed elsewhere on the x-axis so that all points a distance c from the origin have zero total electric potential. What is that additional charge and where is it located? Express your answer in terms of q, c, and b.



(contributed by Mary Mogge)



A3. Young's Triple Slit Coherent light of wavelength λ passes through three infinitesimally narrow slits with spacing d. It produces an interference pattern on screen a large distance $D >> d$

away. (The figure above has been distorted to fit the page.) Assume the light through all three slits is in phase at the location of the slits. Further assume that a wave through any slit has amplitude A at any point P on the screen.

a. (3) What is the phase shift between adjacent waves arriving at point P as a function of θ?

b. (3) Write a wave function for each of the three waves arriving at point P. Specify which wave is which by using the subscripts "t", "m", and "b" to denote the wave through the top, middle, and bottom slit, respectively. Define any quantities you introduce.

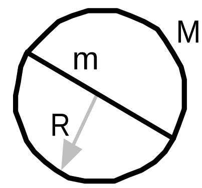
c. (7) What is the amplitude of the resultant wave at point P?

d. (3) Write an expression for the intensity on the screen as a function of angle θ. Let I_o be the intensity at $\theta = 0$.

e. (4) What is the position of the first minimum, that is, what is the smallest angle at which the intensity is zero?

(contributed by Mary Mogge)

A4. A pendulum bob is constructed by taking a thin, uniform-density circular ring of mass M and radius R and affixing a straight, thin, uniform density rod of mass m and length $2R$ across its diameter as shown in the diagram. The pendulum hangs in a vertical plane from a frictionless pivot that can be attached to the ring at any point. The pivot allows the bob to swing either in the plane of the bob or in the plane perpendicular to the bob. Assume the angular amplitude is small.



a. (5) What swinging configurations give the maximum period?

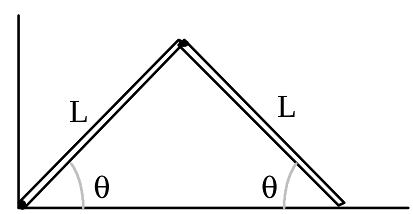
b. (5) What swinging configurations give the minimum period?

c. (10) Find the ratio of the maximum period to the minimum period.

(contributed by Leaf Turner)

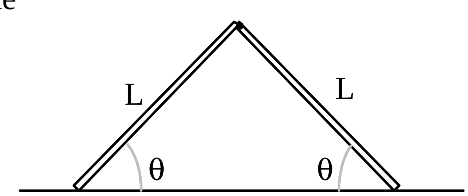
Part B

B1. Consider two uniform rods each of mass M and length L, hinged together to form an upside-down "vee" shape whose angle can vary. The rods are released from rest when their angle θ with the horizontal is 45^o. All hinges are frictionless and have negligible mass.



Case 1

Case 1: The left end is also hinged at a fixed point at the bottom. The right-hand end of the configuration slides on the horizontal surface without friction.



Case 2

Case 2: Both ends can slide frictionlessly along the surface.

(The sub-parts of this problem may be worked in any order.)

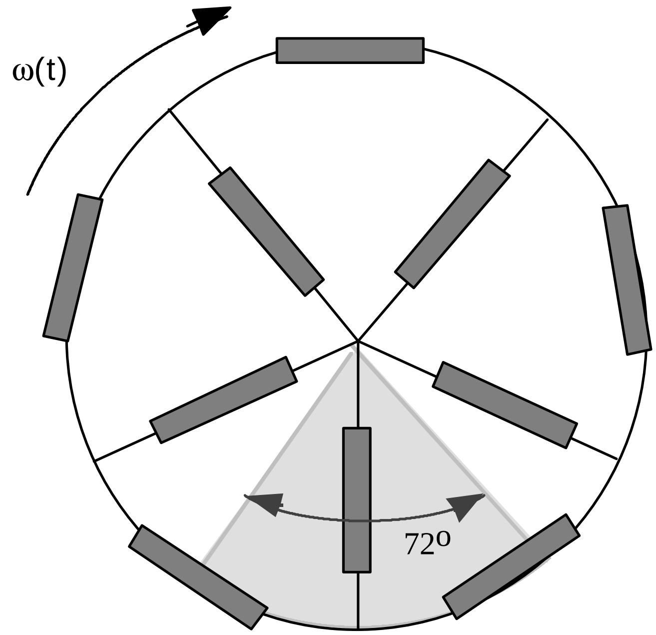
(10) a. Find the upward force F_N that the horizontal surface exerts on the right hand side of the Case 1 rods just after release, when $\theta = 45^o$.

(10) b. Repeat Part (a) for Case 2.

(10) c. Find the angular velocity of the Case 1 rods as a function of θ. $0 < \theta < 45^o$

(10) d. Repeat Part (c) for the configuration of Case 2.

(contributed by Leaf Turner)



A magnetic field ***B***, directed into the page, exists in the shaded region.

B2. (40) Consider the following model of the effect of eddy currents. A rigid wheel, shown in the diagram to the right, consists of ten identical resistors each having resistance *R*. Five of the resistors are equally spaced around a circumference of radius r_o. The other five resistors form spokes of the wheel with an angle of $72^o = 2\pi/5$ radians between adjacent spokes. Assume the resistors have negligible thickness and follow the curvature of the circumference. The wheel rotates about a fixed axis through a circular-sector-wedge-shaped uniform magnetic field ***B***. The sides of the wedge have a length r_o and the angle of the wedge is 72^o. The vertex of the wedge coincides with the axis of the rotating wheel. The wheel's moment of inertia about its center of mass is I_o. At time *t*, the wheel's angular velocity is $\omega(t)$.

(15) a. Draw a diagram of the wheel which shows the direction and magnitude of the electric current in each of the ten resistors. Express all currents on the diagram as a multiple of the smallest current *I*. What is *I*?

(5) b. Find the total power resistively dissipated by the wheel.

(10) c. Find the angular acceleration of wheel and indicate its direction.

(10) d. Now suppose there are nine equiangular $40^o = 2\pi/9$ radian sectors and eighteen identical resistors each having resistance R. Nine resistors are on the spokes and nine are on the circumference between each pair of spokes. The angle of the magnetic field wedge is also reduced to 40^o. Repeat Part (a) for this eighteen resistor case.

(contributed by Leaf Turner)

The 1999 Semi-Final Examination questions were written by the coaches of the United States Physics Team. The coaches are: Academic Director, Dr. Mary Mogge – Professor of Physics at California State Polytechnic University, Pomona, CA; Senior Coach, Dr. Leaf Turner – Physicist in the Theoretical Division of Los Alamos National Laboratory, Los Alamos, NM; Dr. Warren Turner – Physics Teacher at St. Paul's School, Concord, NH. The coaches would also like to thank former academic director, Dr. Larry Kirkpatrick for his many helpful comments.